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Numerical investigations of heat transfer from impinging annular jet

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Abstract

Numerical investigations were performed to predict heat transfer characteristics of laminar annular jets impinging on a surface. An axi-symmetric formulation was used for solving the mass, momentum and energy equations with SIMPLE algorithm. The performance of an annular jet was compared with a standard circular jet having the same values of mass and momentums efflux at the nozzle exit. It was found that heat transfer from the annular jet was about 20% less compared to the circular jet. The distribution pattern of the Nusselt number over the impinging surface scales with $Re^{0.55}$. The paper also discusses the importance of defining appropriate characteristic length for the annular jet.

Keywords: Heat transfer; Impingement; Numerical methods; Annular

1. Introduction

Impinging jets are widely used for heating, cooling and drying applications. Applications of them are found in metals manufacturing, paper and textile industries, gas turbine combustors and food processing. The jets are generally issued from a nozzle with a circular sections or rectangular slots and accordingly identified as circular jet and slot jet, respectively. In combustion systems, two streams of fluids are issued from two concentric sections. The outer portion forms an annular jet. A single jet could also be issued from an annular section. While a lot of information is available on circular jets, little attention has been paid to the heat transfer characteristics of such annular jets. It may be mentioned here that the transport properties over a surface could be controlled effectively by vectoring the jets [1–3].

Earlier studies on annular jets include the work of Maki and Yabe [4] and Maki and Ito [5]. Maki and Yabe [4] performed experiments on annular turbulent jets. They have observed that depending upon the jet height, reverse stagnation point may occur in the flow field. The heat transfer at the reverse stagnation point had a weak dependency on the Reynolds number. The characteristic length of the jet was the jet width. While, analytical and numerical works on liquid annular jet were reported by Ramos [6,7], Trávíčnek et al. [8] reported the application of annular jets in fluidic control. In the open literature, information is lacking on the performance of laminar annular jets and their relative strength vis-à-vis axial jets.

In the present study, numerical experiments were performed for an annular jet in the laminar regime. The heat transfer from the annular jet is compared with the circular jet at the same value of Reynolds number (Re) while keeping the mass and momentum efflux at the nozzle exit at the same level.

To achieve the above objective it is very important to define the characteristic length. In the earlier works [4,5], two times the width of the annular jet was assumed as the characteristic length for defining Re. In that case, for the same value of Re, annular jets have higher mass and momentum efflux at the nozzle exit. This is because if w is the width (i.e. the difference of outer and inner radius) of an annular jet, the area of annular jet is

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Nomenclature

C_f	skin friction coefficient
d_i	inner diameter of nozzle
H	height of the computational domain
h	nozzle-to-plate spacing
h^*	heat transfer coefficient
L	length of domain in r direction
L_c	characteristic length
Nu	Nusselt number ($h^* L_c/\kappa$)
Nu_{av}	average Nu
Nu_c	Nu_{av} for circular jet
Nu^*	Nu_{av} of an annular jet when Re is based on 2×jet width.
p	pressure
Pr	Prandtl number
r	radial coordinate
r_i	inner radius of annular nozzle
Re	Reynolds number ($w_{in} L_c/\nu$)
T	temperature
u	velocity component in radial direction
w	velocity component in axial direction
w_{in}	nozzle exit velocity
z	axial coordinate

Greek symbols

ν	kinematic viscosity
τ	shear stress
κ	thermal conductivity

Subscripts

∞	ambient
av	average
in	inlet (at nozzle exit)
s	impinging surface

$\pi \times ((r_i + w)^2 - r_i^2)$, where r_i is the inner radius. The simplified value of this area is $\pi(2r_i w + w^2)$ which is always grater than the area of a circle with radius w. Thus, for the same inlet velocity at nozzle exit, the annular ring with width w produces higher mass flux compared to the circular area with radius w. Therefore, it would be logical to define Re with the inner diameter of the nozzle as characteristic length and choose the outer diameter in such a way so that the mass and momentum efflux at the nozzle exit is at the same value as in the case of a circular jet.

Fig. 1 shows the axi-symmetric portion of an annular jet. In the present study, we compare the performance of an annular jet of inner diameter d_i with a circular jet of diameter $d = d_i$. The jet height h is two times the characteristic length. The length of the computational domain was $5d_i$ and the height of confinement plate H was $5r_i$. It was established in literature that such a value of computational length is sufficient to eliminate the end effect [2]. The outer diameter of the annular portion is given by $\sqrt{2}d_i$ so that the area under the annular portion is same as the area of a circle with diameter d_i. This also ensures for the same value of jet velocity, the momentum efflux at the jet exit is equal to that of a circular jet.

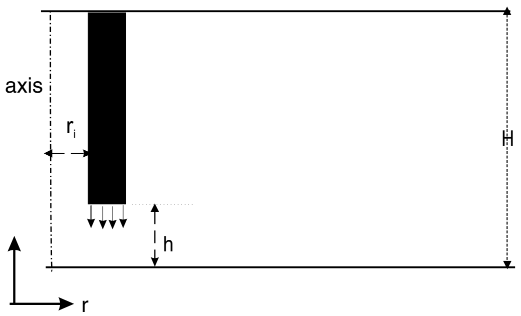

Fig. 1. Schematic of an annular jet.

2. Mathematical formulation

It was assumed that the flow is incompressible with constant properties. The working fluid was air with $Pr = 0.71$. The steady flow field is described by the continuity, momentum and energy equations in cylindrical polar coordinates. The non-dimensional equations with rotational symmetry are provided below:

Continuity equation

$$\frac{1}{r}\left(\frac{\partial(ru)}{\partial r}\right) + \frac{\partial w}{\partial z} = 0. \tag{1}$$

Momentum equation in radial direction

$$u\frac{\partial u}{\partial r} + w\frac{\partial u}{\partial z} = -\frac{\partial p}{\partial r} + \frac{1}{Re}\left(\nabla^2 u - \frac{u^2}{r}\right). \tag{2}$$

Momentum equation in axial direction

$$u\frac{\partial w}{\partial r} + w\frac{\partial w}{\partial z} = -\frac{\partial p}{\partial z} + \frac{1}{Re}\nabla^2 w. \tag{3}$$

Energy equation

$$u\frac{\partial T}{\partial r} + w\frac{\partial T}{\partial z} = \frac{1}{Re \cdot Pr}\nabla^2 T. \tag{4}$$

For non-dimensionalization, the characteristic length and velocity is d_i and w_{in}, respectively. The pressure and temperature has been non-dimensionalized by $p = p/2\rho w_{in}^2$ and $T = (T - T_\propto)/(T_w - T_\propto)$, respectively.

As the simulation was performed in an axi-symmetric domain, at the axis of symmetry, i.e. $r = 0$, gradients of all primitive variables are zero. At the exit plane, i.e. $r = 5d_i$, a convective boundary condition [9] has been used. The non-dimensional temperature of the impinging plate was taken as unity (i.e. $T_s = 1.0$) and the jet temperature was assumed at the ambient level (i.e. $T_{in} = 0.0$).

The governing equations were solved using the SIMPLE formulation [10] in a collocated grid array. Convective terms have been discretized by a weighted average of second order up-wind method and the central difference scheme. For the diffusive terms second order central differencing scheme has been used.

3. Results and discussion

Computations have been performed using 100×200 grids in an HP workstation. Studies with different grid resolutions showed that the average Nu obtained with the present grid differed from the grid independent solution by less than 2%. The results for the case of standard circular jet matched well with the results of Owsenek et al. [11].

The skin friction coefficient, C_f, is given by

$$C_f = \frac{\tau_s}{\frac{1}{2}\rho w_\infty^2},$$

where τ_s is the shear stress on the impinging surface. Under the present scheme of non-dimensionalization, the term can be finally calculated as the gradient of velocity, i.e., $C_f Re = \frac{\partial u}{\partial z}$.

The heat transfer performance is measured by Nusselt number which can be calculated by the local temperature gradient as $Nu = -\frac{\partial T}{\partial z}$. The average Nusselt number can be obtained by $Nu_{av} = -Nu(r)\partial r/L$, where L is the length of the computational domain in r direction, i.e. $5d_i$.

In Fig. 2, the comparison of the distribution of the skin friction coefficient between a circular jet and an annular jet has been presented. It can be discerned here that for the circular jet, the value of C_f initially increases up to the value of $r = 0.8$ and then reduces monotonically. In case of an annular jet, C_f is zero at the axis $(r = 0)$ and then it reduces up to $r = 0.6$, beyond which it starts increasing. At about $r = 1.0$, the value of C_f is again zero after which the value is in positive range. It reaches a peak at about $r = 1.2$ and then reduces. The values of C_f beyond $r = 3.0$ are identical for both types of jet. Thus the major difference for the annular jet is that it forms a zone of negative skin friction. In Fig. 3, distribution of C_f at different Re is presented. It can be observed that the profiles are self-similar.

The difference of heat transfer distribution between the annular and the circular jet is depicted at Fig. 4. The

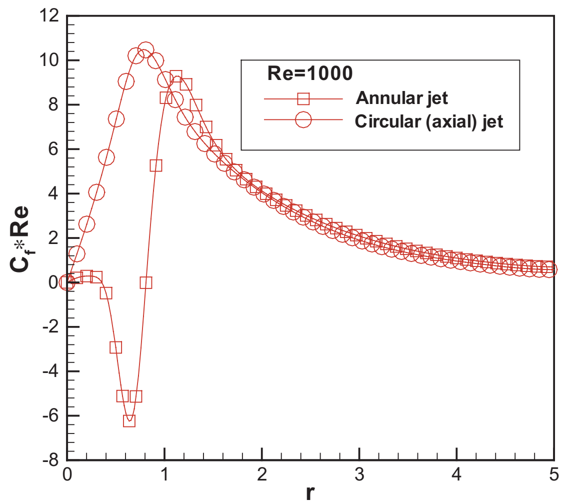

Fig. 2. Comparison of C_f between the annular and circular jet (location of annular jet at $r = 0.5$–0.707).

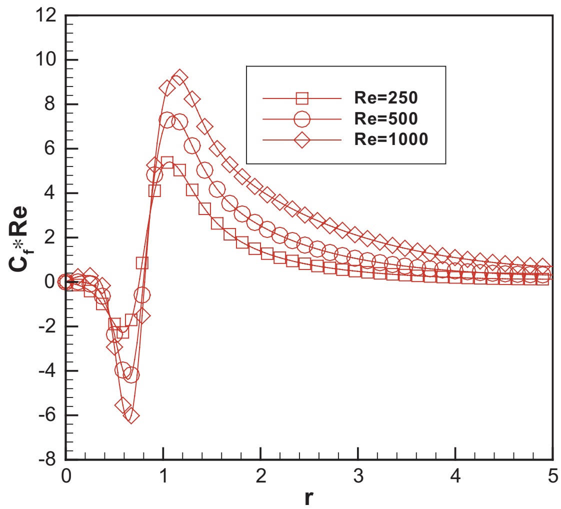

Fig. 3. Distribution of C_f at different Re for an annular jet.

peak heat transfer for a circular jet occurs at slightly downstream of the stagnation region and then the value of Nu reduces monotonically. However, for the annular jet, the peak value of Nu occurs at about $r = 0.9$ which is downstream of the annular jet. The value of Nu reduces on both the left and right sides of the peak value. The location of peak heat transfer is again at downstream from the stagnation region.

The dependence of heat transfer from annular jets on Re is plotted in Fig. 5. The distribution profiles are self-similar at all Re. The peak heat transfer occurs at about $r = 0.9$ for all values of Re. However, as Re increases, the

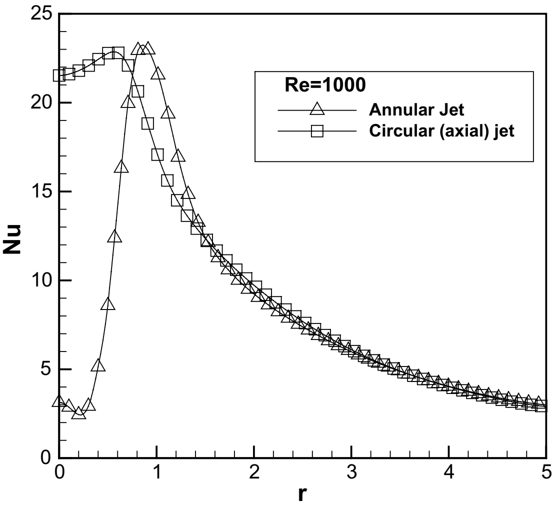

Fig. 4. Comparison of *Nu* for the annular and the circular jet.

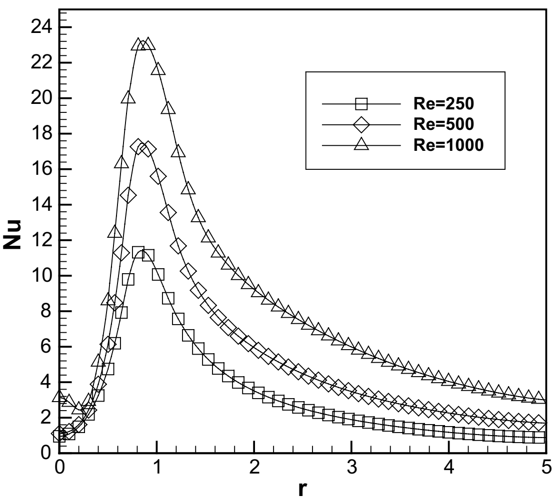

Fig. 5. Distribution of *Nu* at different *Re* for an annular jet.

location of peak *Nu* shifts further downstream. The value of *Nu* at the axis ($r = 0$) is always lower than the asymptotic value at the downstream side. By analyzing the data, it was found out that the distribution of *Nu* for annular jet scales with $Re^{0.55}$. It can be mentioned here that *Nu* for the axial jet and the knife-jet also scales with $Re^{0.5}$ and $Re^{0.55}$, respectively [12].

The results of the investigations are summarized in Table 1. From this table, it is observed that for the same amount of the mass and momentum flux at the jet exit, heat transfer from the annular jet is lower than that of a circular jet. For example, Nu_{av} values at $Re = 250$ were 4.49 and 3.34 for circular and annular jets, respectively. However, as *Re* increases, the reduction in heat transfer

Table 1
Heat transfer results at a glance

Re	Nu_c	Nu_{av}	Difference (%)	Nu^*
250	4.49	3.34	25.6	5.14
500	6.88	5.41	21.4	7.65
1000	10.16	8.17	19.6	10.96

Nu_{av}, average *Nu* for annular jet; Nu_c, Nu_{av} for circular jet; Nu^*, average *Nu* for annular jet with *Re* based on double the jet width.

reduces. This table also provides the results showing the value of Nu^* where the outer diameter of the annular jet is equal the inner diameter. Here two times width of the annular portion is the characteristic length for defining *Re* and the heat transfer from the annular jets is however higher. But it is to be borne in mind that in such a situation, the mass flow is higher compared to the circular jet.

4. Concluding remarks

Heat transfer from annular laminar jet with the simplest configuration has been numerically investigated. For the same amount of mass and momentum efflux at the nozzle exit, *Nu* of annular jets are about 20% lower compared to the circular jets. The characteristic length for defining *Re* should be the inner diameter of the annular ring and width is to be chosen as $(\sqrt{2} - 1)$ times the inner radius. This ensures that the area of the annular portion is equal to the area of a circle with the diameter as characteristic length.

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